A New Variation of the Rotation-by-Magnetization Method of Measuring Gyromagnetic Ratios*

C. J. DAVISSON AND J. W. BEAMS
University of Virginia, Charlottesville, Virginia

I. INTRODUCTION

ALL of the experimental arrangements which have so far been used to measure gyromagnetic ratios by the rotation-by-magnetization method have had certain features in common. The specimen has in all cases been a delicately suspended rod mounted on the common axis of one or more vertical solenoids. The experimental procedures, on the other hand, have been various. The first to be tried, though not the first to yield publishable results, was the ballistic, that in which the reaction of the rod to the change in internal angular momentum, consequent to a sudden change in its magnetization, is measured by the angular throw of the rod against the tortion of its suspension. An investigation of this kind was started by Richardson[1] at Princeton in 1908, the same year in which he predicted, on the basis of the electron theory, the existence of the gyromagnetic effect and derived a value for the gyromagnetic ratio. This line of work was continued at Princeton for ten years and culminated in the accurate measurements of gyromagnetic ratios made by J. Q. Stewart[2] in 1918. This was five years after Richardson had returned to England. The work had been continued under the direction of the late H. L. Cooke.

In the meantime (1915) Einstein and de Haas[3] had verified Richardson's prediction and had measured gyromagnetic ratios. (The same feat was performed in the same year by S. J. Barnett by the converse magnetization-by-rotation method with which we are not here concerned.) Einstein and de Haas employed a resonance method in which the magnetizing field was periodic and tuned to the natural frequency of the rod and its suspension. Values of the ratio were deduced from the amplitude of the vibration and other relevant dimensions of the system.

A third experimental procedure has been devised and perfected more recently by Chattock, Sucksmith and Bates.[4,5] As in the Einstein-de Haas method the magnetizing field is periodic, but the torque on the rod resulting from the gyromagnetic effect is countered by an equal and opposite torque derived from the varying field of the magnetized rod. It is a beautiful and ingeneous technique capable of precise measurements as its inventors have amply demonstrated.

There have, thus, been three ways in which the rod and filament experiment has been carried out—the ballistic method used by Stewart; the Einstein and de Haas resonance method; and the null method of Chattock, Sucksmith, and Bates. It is probable that variations on the rod and fiber theme have been exhausted, but by making some rather drastic alterations in the experimental arrangement, yet another way of measuring gyromagnetic ratios emerges. It is this new variation which is the subject of this communication.

II. THE NEW EXPERIMENTAL ARRANGEMENT

In the new arrangement the rod is replaced by a sphere, mechanical suspension is replaced by tortionless magnetic suspension, and solenoids of the usual kind are replaced by a pair of air-cored electromagnets mounted on a common vertical axis. The point of suspension, which is in vacuum, is at the center of symmetry of this pair of coils, as shown schematically in Fig. 1. Lateral stability is provided over a wide range of operating conditions by the form of the field. Longitudinal quasi stability is achieved by automatic control of the current in the upper coil. This is the rectified output of a rather elaborate vacuum tube circuit (Fig. 4) which operates at radio frequency. The control is in accordance with signals received in the circuit from a small pick-up coil located a short distance below the sphere with which it has a loose variable inductive coupling. This servomechanism was devised some years ago by one of us and his students[6] and has been developed to a high degree of dependability.

The weight mg of the magnetically suspended sphere is balanced by the upward force exerted on it by the field. This, to an exceedingly close approximation, is $M(dH/dz)=MH'$, the dipole magnetic moment of the sphere multiplied by the gradient of the magnetic field intensity H at its center:

$$MH'=mg. \tag{1}$$

The coil system is symmetrical with respect to a horizontal plane through the point of support. Consequently, the intensity H and all even derivatives of H are proportional to the sum of the currents in the two

* This work was supported by the Bureau of Ordnance, U. S. Navy, Contract NOrd-7873 at the University of Virginia.

[1] O. W. Richardson, Phys. Rev. **26**, 248 (1908).
[2] J. Q. Stewart, Phys. Rev. **11**, 100 (1918).
[3] A. Einstein and W. J. de Haas, Verhandl. deut. physik. Ges. **17**, 152 (1915).
[4] A. P. Chattock and L. F. Bates, Trans. Roy. Soc. (London) **A223**, 257 (1923).
[5] W. Sucksmith and L. F. Bates, Proc. Roy. Soc. (London) **104A**, 499 (1923).
[6] Beams, Young, and Moore, J. Appl. Phys. **17**, 887 (1946); J. W. Beams, Wash. Acad. Sci. **37**, 221 (1947); J. W. Beams, Rev. Sci. Instr. **21**, 182 (1950).

coils, and all odd derivatives are proportional to the difference of these currents,

$$\begin{aligned} H&=k_0(J_1+J_2),\\ H'&=k_1(J_1-J_2),\\ H''&=k_2(J_1+J_2), \quad \text{etc.,} \end{aligned} \tag{2}$$

where J_1 and J_2 represent the current densities in the upper and lower coils, respectively. The constants k can be calculated from the dimensions of the coils and their separation.

By virtue of relations (2) H and H' are capable of independent variation. A change in M produced by a change in H can be coordinated with a change in H' which preserves the relation (1) and so leaves the position of the sphere unaltered. The sphere can be taken from a state 1 in which its magnetic moment is $M_1=mg/H_1'$ to a state 2 in which its moment is $M_2=mg/H_2'$ while it is held constantly at or near the center. The change in magnetic moment,

$$(M_2-M_1)=mg/(1/H_2'-1/H_1'), \tag{3}$$

is accompanied, through the gyromagnetic effect, by a proportionate observable change in the angular momentum of the sphere about its vertical axis. The angular momentum is changed from some initial value Iw_1 to some final value Iw_2, where $I(=(2/5)mr^2)$ is the moment of inertia of the sphere and w is its angular velocity. The change in angular momentum is

$$(2/5)mr^2(w_2-w_1), \tag{4}$$

and this divided by the change in magnetic moment (3) is, by definition, the gyromagnetic ratio ρ of the substance of the sphere, so that

$$\rho=[2r^2(w_2-w_1)]/[5g(1/H_2'-1/H_1')]. \tag{5}$$

This is the fundamental equation of the new scheme of measurement.

The evaluation of (5) presents no great experimental difficulty. All that need be known about the sphere is its radius. Means must be provided for measuring accurately the uniform angular velocities w_1 and w_2, and for measuring the currents in the coils at the beginning and at the end of the transition. The values of H' are computed from the second of Eqs. (2) rewritten in the form

$$H'=k_1N(i_1-i_2), \tag{6}$$

where i_1 and i_2 are the currents corresponding to J_1 and J_2, and N is the number of turns per unit area in the windings.

III. BEHAVIOR IN DETAIL

To examine the behavior of the system more closely we consider the special case in which the permeability of the sphere is infinite. The magnetic moment acquired by such a sphere in a field of intensity H is

$$M=r^3H. \tag{7}$$

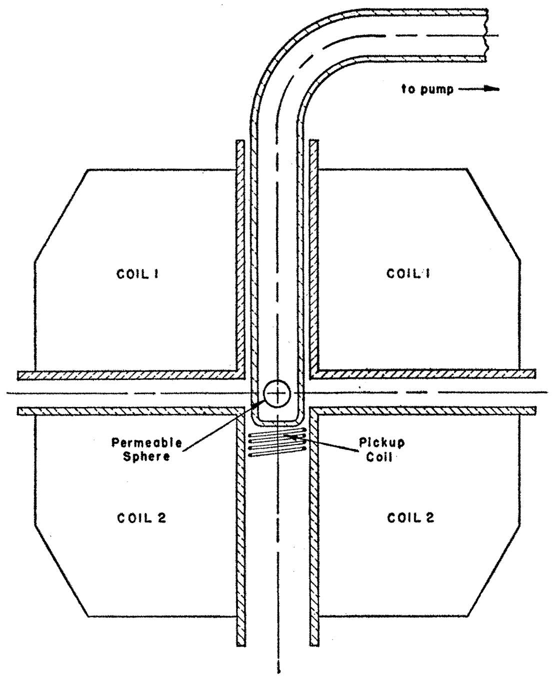

FIG. 1. Schematic diagram of experimental apparatus.

This is only slightly greater than the moment the sphere would acquire if its permeability instead of being infinite were merely high, say 10^4 or as low even as 10^3. The assumption is not, therefore, so very unrealistic.

Substituting this expression for M into (1) one obtains

$$HH'=(m/r^3)g=(4\pi/3)dg, \text{ a constant,} \tag{8}$$

as the field condition for the magnetic suspension of a highly permeable sphere of density d. HH' is a constant proportional to the density of the sphere but independent of its size. For iron the constant is 3.23×10^4.

Substituting for H and H' the expressions for these quantities given in Eqs. (2) the condition for magnetic suspension in terms of the current densities in the two coils is found to be

$$J_1^2-J_2^2=\frac{4\pi dg}{3k_0k_1}=J_0^2, \text{ a constant.} \tag{9}$$

J_0 is the current density in the upper coil when the sphere is supported by this coil alone. It is the same for spheres of all sizes. Of the two branches of the hyberbolic relationship (9) only one need be considered, the natural choice being that for which J_1 is positive. A central portion of this characteristic is plotted in Fig. 2. The part available for experimental observations is limited, in the first place, by the current-carrying capacity of the coils. If $\bar{J}$ represents the highest current density at which either of the coils can be safely operated, then the observations are restricted to the

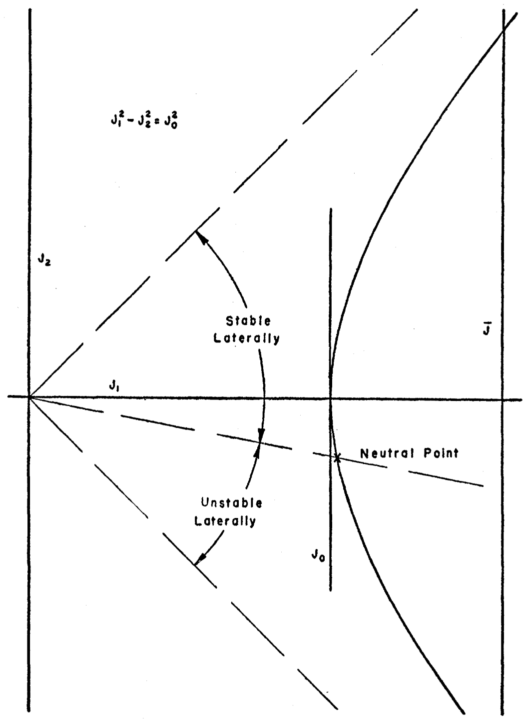

FIG. 2. J_1, J_2 characteristic of magnetically suspended sphere of high permeability.

range in which J_1 is not greater than $\bar{J}$. The coils must, of course, be so designed that J_0 is definitely less than $\bar{J}$.

When $J_1 = \bar{J}$, $J_2 = \pm(\bar{J}^2 - J_0^2)^{\frac{1}{2}}$, and by the first of Eqs. (2),

$$H = k_0[\bar{J} \pm (\bar{J}^2 - J_0^2)^{\frac{1}{2}}]. \tag{10}$$

The larger of these values of H is the magnetizing intensity to which the sphere is exposed when at the upper end of the observational range, and the smaller that to which it is exposed when at the lower end. Designating these H_2 and H_1, respectively, the range of values of H allowed by the current-carrying capacity is

$$H_2 - H_1 = 2k_0(\bar{J}^2 - J_0^2)^{\frac{1}{2}}. \tag{11}$$

By (7) the corresponding range of magnetic moment is

$$M_2 - M_1 = 2k_0 r^3 (\bar{J}^2 - J_0^2)^{\frac{1}{2}}. \tag{12}$$

The change in angular velocity is

$$w_2 - w_1 = \rho(M_2 - M_1)/I, \tag{13}$$

where ρ represents the gyromagnetic ratio of the sphere and I its moment of inertia. Writing $I = (8\pi/15)dr^5$ and using (12), the expression for the change in velocity becomes

$$w_2 - w_1 = \frac{15k_0\rho}{4\pi d}\frac{1}{r^2}(\bar{J}^2 - J_0^2)^{\frac{1}{2}}. \tag{14}$$

Other circumstances being the same, the change in angular velocity is inversely proportional to the square of the radius which, as it turns out, puts a premium on using as small a sphere as possible.

Equation (14) is an expression for the change in angular velocity which could be produced in an infinitely permeable sphere if the range of observation were limited by the current-carrying capacity of the coils alone. There is, however, another limitation.

A necessary condition for observing changes in velocity is, naturally, that the sphere remain on the axis—that it have lateral stability. There is always some part of the characteristic hyperbola (Fig. 2) over which this condition is not met. This may overlap the otherwise available range of observation, or even engulf it completely, depending on the design of the coils. The condition for lateral stability is that the quantity

$$F = (H'^2 - 2HH''), \tag{15}$$

which is a factor in the lateral force constant, be negative. In terms of the current densities

$$F = k_1^2(J_1 - J_2)^2 - 2k_0k_2(J_1 + J_2)^2. \tag{16}$$

When F is equal to zero the equilibrium is neutral. In this case (16) can be reduced to read

$$\frac{J_2}{J_1} = \frac{k_1 \pm (2k_0k_2)^{\frac{1}{2}}}{k_1 \mp (2k_0k_2)^{\frac{1}{2}}}. \tag{17}$$

There will be no real ratio J_2/J_1 for which the equilibrium is neutral, and, consequently, no part of the hyperbola over which the equilibrium is stable, unless $(2k_0k_2)$ is positive, and this is not necessarily the case. We assume that the coil has been so designed that the condition is satisfied. Equation (17) then represents two straight lines through the origin of the J_1, J_2 diagram, Fig. 2. One, and only one, of these intersects the hyperbole. There is, thus, only one pair of values J_1 and J_2, for which the sphere is stable longitudinally and neutral laterally. The part of the hyperbola on one side of this unique point is the region in which the lateral equilibrium is stable; the part on the other side is that in which the equilibrium is unstable. It can be argued from (9) and (16) that the stable region is that *above* the neutral point. The part of the hyperbola available for observations is, thus, that portion of the range allowed by current-carrying capacity which lies above the neutral point. This may be all, a part only, or none of this latter range, depending on the design of the coils and their separation.

The case here examined is that in which the permeability of the sphere is 10^3 or greater for all available values of H. If the permeability is low, but independent of H, the characteristics are the same as in the case of high permeability except for an increase in the value of J_0. As the permeability decreases J_0 approaches $\bar{J}$, the observational range becomes shorter, and at some permeability greater than 1 vanishes. The increase in J_0 with lowering permeability is not, however, as marked as might be expected. J_0 for permeability 10, for example, is only 15 percent greater than for permeability infinite. If the permeability is not high and not independent of H, the characteristic is no longer a rectangular hyperbola, and if the specimen is hysteretic the characteristic is no longer single valued in J_1. But neither of these circumstances, nor the presence of residual magnetization, have any effect on the validity of Eq. (5) on which the measurement of gyromagnetic ratios is based.

IV. APPARATUS

(1) The Coil System

Parts of the coil system which have been built to try the new method of measuring gyromagnetic ratios are shown in cross section in Fig. 3. Described in polar coordinates with the center of the system as origin, the outer boundary of the coil is the surface

$$r=(27/4)^{\frac{1}{4}}\bar{r}\sin\theta\cos^{\frac{1}{2}}\theta, \tag{18}$$

where $\bar{r}$ = maximum extension = 15.75 cm.

The inner boundaries are

(a) a cylinder, $r=a/\sin\theta$, where $a=1.27$ cm, (19)

and

(b) a plane, $r=(1/2)^{\frac{1}{2}}a/\cos\theta$. (20)

The constant k_1 of the second of the Eqs. (2) is greater for this coil than for any other having the same inner boundaries and the same cross-sectional area. It was thought, erroneously, that coils of this form would produce the greatest change in the angular velocity of a given sphere. The error was recognized while the coils were being built, but the work was carried on to completion nevertheless.

The frame on which the coils are wound is of brass and is accurately machined. The central member is a 1-inch tube with $\frac{1}{8}$-inch walls. The plates which maintain the plane boundaries are $\frac{1}{8}$ inch thick and are separated by spacers—a central ring spacer integral with the tube and others further out. Ports cut through the ring spacer and tube afford views of the sphere from four directions. Fixed and temporary forms were used to give the coils the prescribed form. The wire is No. 25 (USG) plain enamel copper. The winding was done on a lathe with a counter to keep track of the turns. The finished structure, including end-plates not previously mentioned, is bolted together on four $\frac{1}{2}$-inch rods. Thin outer cylinders attached to the appropriate plates complete a reservoir about each of the coils, and these, when the structure is mounted, are filled with transformer oil—otherwise there is sparking.

The constants of the first three of the Eqs. (2), found by appropriate integrations over the cross section, are

$$k_0=4.54,\quad k_1=1.10,\quad k_2=0.313.$$

The number of turns per cm^2, N, is 408.

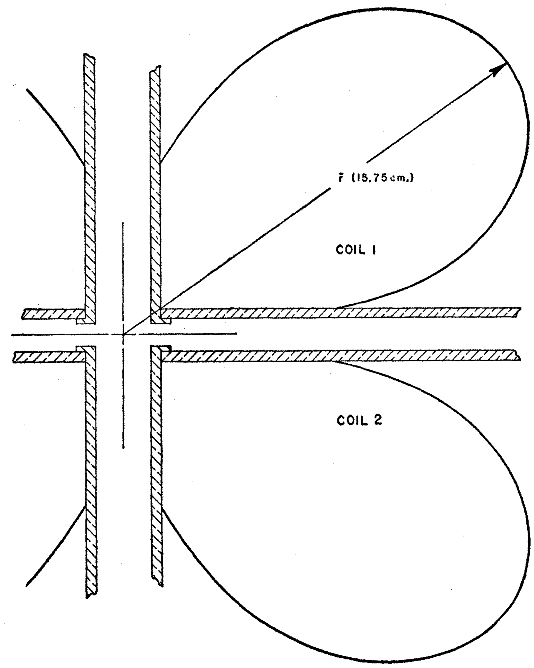

FIG. 3. Cross section of experimental coil system.

Substituting these values of the k constants into (17) one finds that the lateral equilibrium should be neutral when

$$J_2/J_1=-0.19.$$

The line which this equation represents in Fig. 1 intersects the hyperbole at the point marked X. The part of the hyperbola available for observation is from this point upward to the limit set by the current-carrying capacity of the coils. The range includes the point J_0, 0 at which the sphere is supported by the upper coil alone. Substituting in Eq. (9) one finds for iron

$$J_0=80.4\ \text{amp/cm}^2.$$

For this value of J_0 the coordinates of the neutral point are

$$J_1=81.2\ \text{amp/cm}^2,$$
$$J_2=-15.6\ \text{amp/cm}^2.$$

Assuming the current-carrying capacity to be 125 amp/cm (it is known to be somewhat higher than this) the coordinates of the upper limit of the range of observations are

$$\bar{J}=J_1=125\ \text{amp/cm}^2,$$
$$J_2=(\bar{J}^2-J_0^2)^{\frac{1}{2}}=95.7\ \text{amp/cm}^2.$$

Equations (2) for H and H' can now be evaluated for the three J_1, J_2 points just considered. The results are given in the following table.

The total available change in H is 694 oersteds. Less than 10 percent of this change occurs in the part of the observational range in which J_2 is negative. Not much is gained by entering this part of the range, particu-

TABLE I. Field characteristics.

J_1 amp/cm^2	J_2 amp/cm^2	H oersteds	H' oersteds/cm
125	95.7	1002	32.2
80.4	0.0	365	88.4
83.4	−15.8	308	107

larly as it is advisable to keep well clear of the region of lateral instability. As a rule, observations are terminated at the point at which $J_2=0$.

(2) The Control Circuit and Vacuum System

A diagram of the control circuit which regulates the current through the supporting solenoid or coil 1 is shown in Fig. 4. It is similar to circuits used previously for supporting small spheres in a vacuum.[6] The circuit consists of a ten-megacycle tuned-grid tuned-plate oscillator, a cathode follower detector stage, two stages of amplification, another cathode follower circuit from which are derived a signal and its time derivative, a mixer stage which recombines these two signals, another cathode follower, and a power stage whose load is the support coil 1. The impedance of the pick-up coil L_1 is increased or decreased as the sphere moves down or up, and hence the tuning and amplitude of the oscillator depends upon the vertical position of the sphere. As a result, the circuit may be adjusted in such a way that the current in coil 1 is increased if the sphere moves downward and decreased if it rises. These current changes are adjusted so that the sphere is maintained accurately in the desired vertical position without "hunting." It will be observed from Fig. 4 that this is accomplished by introducing a derivative or "anti-hunt" along with the signal from L_1. When the circuit is properly adjusted, the vertical and horizontal stability of the freely suspended sphere is such that no motion can be observed with a 50-power microscope focused on scratches on the sphere. Coil 1 and coil 2 each consisted of 40,308 turns of no. 25 copper wire. The remaining values of the components of the circuit are given in Fig. 4.

The vacuum system is of glass except for a sealing wax joint and a black Bakelite support for the coil L_1. It is evacuated with a standard forepump, diffusion pump cold trap combination. The system contains both a McLeod and an ionization gauge. The drag on the rotating sphere resulting from air friction is given[6] by

$$\log_e N/N_0 = -5p/rd(M/2\pi RT)^{\frac{1}{2}}(t-t_0),$$

where N_0 is the number of rps at the time t_0, N is the number of rps at time t, p is the pressure in dynes/cm^2, d is the density of the sphere, T is the absolute temperature, M is the molecular weight of the gas, r is the radius of the spherical rotor, and R the gas constant. The working pressure is 10^{-6} mm of mercury or less, so for a 0.5 mm diameter sphere $N/N_0=\exp[-4.8\times10^{-7}(t-t_0)]$, which is a very small correction.

V. THE CHANGE IN ANGULAR VELOCITY

To arrive at an estimate of the change in angular velocity that can be produced with the present coil system we rewrite Eq. (5) in the form

$$(w_2-w_1)=5g/2r^2(1/H_2'-1/H_1')$$

and evaluate for the case of a highly permeable sphere taken from state 1 in which $J_1=J_0$ to state 2 in which $J_1=\bar{J}$. As gyromagnetic ratio we take the base value

$$\rho=m/e=1/(1.76\times10^7).$$

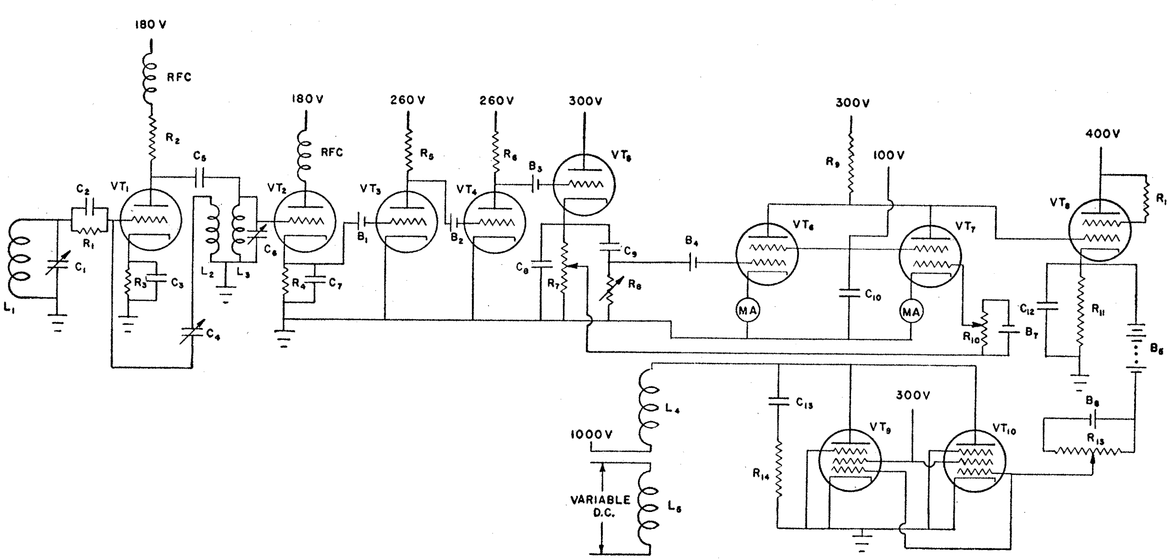

FIG. 4. Control circuit. $R_4=R_5=R_6=R_7=10^5$ ohms; $R_9=R_{10}=R_{13}=5\times10^4$ ohms; $R_{12}=R_{14}=2\times10^4$ ohms; $R_3=2\times10^2$ ohms; $R_1=8\times10^3$ ohms; $R_2=7.5\times10^4$ ohms; $R_{11}=5\times10^3$ ohms. $C_5=C_7=C_8=0.004$ mf; $C_{10}=C_{13}=0.5$ mf; $C_2=0.0025$ mf; $C_3=0.01$ mf; $C_9=0.1$ mf max (variable); $C_{12}=0.05$ mf. $VT_1-VT_2-VT_3-VT_4-VT_5-6J5$; VT_6-VT_7-6SJ7; VT_8-6L6; $VT_9-VT_{10}-813$. $B_1=28$ v; $B_2=157.5$ v; $B_3=90$ v: $B_4=9.5$ v; $B_7=45$ v; $B_5=45$ V; $B_6=180$ v.

Using the values of H' given in Table I we find that for

$$2r=1.0\text{ mm}=0.1\text{ cm}\quad (w_2-w_1)=1.08\times10^{-3}\text{ rad/sec}=3.89\text{ rad/hr},$$

$$2r=0.5\text{ mm}\quad (w_2-w_1)=15.6\text{ rad/hr}=2.48\text{ rphr},$$

$$2r=0.25\quad (w_2-w_1)=9.90\text{ rphr}.$$

VI. THE EXPERIMENTAL PROCEDURE

The experiment is begun with the sphere resting on a small platform several mm below the center of the system. The current in the lower coil is raised to about 100 amp/cm². The effect of this is to press the sphere more tightly against the platform. The current in the upper coil is then raised to 100 amp/cm². The sphere is now strongly magnetized but no force is exerted on it by the field, since H' is zero. It rests on the platform only under its weight. Next the current in the upper coil is raised to 110 or 115 amp/cm² and then increased slowly beyond this point. As this is done the point on the axis at which the sphere would be in quasi equilibrium longitudinally approaches the sphere from above. When the distance between the sphere and its equilibrium position has been reduced to about 1 mm the sphere springs upward and is locked in position by the servomechanism which controls the upper coil current. The sphere can now be raised by decreasing J_1 or lowered by increasing J_1. It may be touched back gently onto the platform to arrest any rotation it may have acquired at take-off, if it is desired to measure the increase of angular velocity from the zero value. Brought to the center, the sphere is in state 1 with w_1, zero or negligibly small. Measurements are made of the currents in the two coils to be used later in calculating H_1'.

The sphere is now brought by steps into state 2, that in which $J_2=0$. Decreasing J_1 raises the sphere, as has been mentioned, but decreasing J_2 lowers it. By decreasing first one and then the other of these currents by small amounts, the system is brought into state 2 with the sphere having been held constantly in suspension at or near the center. The current in the upper coil is measured for use in calculating H_2', and measurement is made of the angular velocity of the sphere. So far the velocity measurement has been based on visual observations and timing with a well regulated watch. The sphere is viewed through a low power microscope at magnification ×10. It has not been found necessary to supply the sphere with fiducial marks. There have always been enough tiny specks and fine scratches on the spheres to serve this purpose.

VII. PRELIMINARY EXPERIMENTAL RESULTS

The present apparatus was constructed for the purpose of testing the method and ferreting out the various sources of error rather than for obtaining ultimate accuracy of measurement. As a result, only order of magnitude values have been obtained. Ball bearing steel spheres 0.75 mm, 0.55 mm, and 0.50 mm have been used in the experiment because they are commercially available and are supposed to be spherical and homogeneous. The spheres were stably suspended when the current in the upper coil was varied from 195 milliamperes with the current in the lower coil at zero, to 370 milliamperes when the current in the lower coil was about 300 milliamperes. This current range can be extended by cooling the coils. It is clear from Table I that smaller spheres should be used in order to obtain higher angular velocities, but it is only recently that we have perfected a method similar to that of Bond[7] for making accurate metal spheres down to 0.2 mm in diameter.

Two principal sources of trouble have developed which have greatly retarded the work but which no doubt can be overcome in future experiments. The first is the development of electrostatic charges on the walls of the vacuum chamber and on the small sphere. The effect of these charges while small on the 0.75-mm sphere is very disturbing on smaller spheres. These electrostatic charges are probably the result of several causes including the photoelectric effect produced by the light which illuminates the rotor and walls of the vacuum chamber, the friction of the rotor on the bottom of the vacuum chamber, as well as the friction of air currents around the vacuum chamber generated by heating of the coils. The effect has been markedly reduced by light filters (including water), but it may be necessary to use new materials for the vacuum chamber in order to completely eliminate it. The second trouble arises from the fact that a hard steel sphere, usually, is not magnetically homogeneous. This gives rise to a small oscillation about the axis of rotation of a suspended sphere rather than a uniform rotation when an extremely small torque is applied and of course reduces the accuracy of the angular velocity measurements and, if conducting material is very close by, perhaps introduces some drag. Two methods of correcting this difficulty have been found effective. The first consists in spinning the rotor to a high rotational speed and then bringing it to rest before the measurements are made. This reduces the effect, but is difficult to carry out with the present experimental arrangement. The second method consists in heat treating the metal sphere until it is as magnetically "soft" as possible. However, the final solution to this problem will probably be best secured by making the spheres of magnetically soft homogeneous material followed by heat treatment.

Spherical samples have been used in the present experiments because the calculations are greatly simplified. However, suspended long ellipsoids of revolution or rods should give rise to greater angular velocities than spheres with comparable radii. Also, the magnetism can more easily be made homogeneous. These favorable factors no doubt will more than compensate for the more detailed calculations necessary. As pointed

[7] W. L. Bond, Rev. Sci. Instr. 22, 344 (1951).

out previously, results to date are not as reliable as those obtained by other methods referred to above, but they show that the method ultimately should give a precision limited only by the accuracy with which the coils can be wound.

VIII. THEORY AND PRACTICE

In theory, the present scheme of measuring gyromagnetic ratios is simpler and less subject to disturbances than those previously used. The field acts to bring the magnetic axis of the sphere into coincidence with its own axis and, since all diameters of the sphere are mechanical axes, the magnetic and mechanical axes coincide, no matter in what diameter the magnetic axis happens to lie. Thus the ideal experimental conditions, which are realized only at the cost of much time and effort in the rod and fiber experiments, are set up automatically. There is a certain advantage also in having the measurement based on the determination of a uniform angular velocity rather than on the determination of the amplitude of a swing or a vibration.

However, in practice, as has been noted above, the behavior of the system is less simple than was anticipated. The fact that the sphere occasionally flies from the axis to the wall of the enclosure or sticks to the bottom of the vacuum chamber is a serious disturbance which must be eliminated, but it is extraneous and not chargeable to the theory of the measurements. The other unexpected behavior, which has been mentioned—that the angular velocity is not uniform, and may in certain circumstances degenerate to an oscillation—is more fundamental. It is natural to surmise that this is the result of the sphere acting as a compass in the horizontal component of the earth's field. This component has not as yet been compensated, so that this explanation has not been definitely ruled out. It is thought, however, that the trouble lies elsewhere. It is assumed in the theory that the sphere is magnetically homogeneous, that it contains a uniform distribution of permeability. If this condition does not obtain, a disturbance of the kind that is observed may be expected. The sphere may be thought of as having "a center of permeability." In the ideal specimen this is at the center of the sphere, but whether it is at the center of the sphere or not, it is the "center of permeability," and not the center of gravity, that the field holds to its axis. It can be seen, by considering an extreme case of non-uniformly distributed permeability, that, owing to a distorted demagnetizing field, the magnetic axis will make an angle with the axis of the field. The gyromagnetic torque acts about the magnetic axis which remains fixed. A consequence is that as the sphere rotates about its magnetic axis, its center of gravity is displaced vertically. It is this that gives rise to the oscillations. It is an essential condition for the success of the measurements that the specimen be both accurately spherical and magnetically homogeneous. Efforts are being made, as described above, to meet this condition.